

Collective excitations of graphene excitons being in the Bose-Einstein condensate state

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Bose-Einstein condensation of the excitons in graphene is considered. We suggested the model spinor equation for neutral particles with short range interaction described the microscopic graphene excitons dynamic. Using this equation we derived quantum hydrodynamic equations for description of collective properties of excitons in graphene, particularly for the case when excitons being in the Bose-Einstein condensate (BEC) state. The dispersion of collective excitations in graphene excitons BEC is studied. We shown that frequency of collective excitations is proportional to the square root of the wave vector module: $\omega \sim \sqrt{k}$.

Bose-Einstein condensation (BEC) of ultracold atomic gases [1] and excitons in semiconductors [2]- [6] and monolayer of carbon atoms (graphene) [7], [8], [9] are very interesting and promising physical systems for present day. In this paper we present model for excitons in graphene and study they properties in the case one's is in the BEC state. In connection with it we make some notice about kinematic and dynamic properties of particles in various fundamental models.

Quantum mechanics and special relatively gives us a new way of the kinematic properties description in comparison with the classic physics. However, the methods of interaction account remains the same as in classic physics. For example, in the Schrodinger equation the Coulomb interaction is considered as interaction of point like particles [10]. Some times the Coulomb interaction is also used in the many-particle generalization of the Dirac's equation [11].

In graphene we have deal with the electrons and holes conductivity. They properties crucially depend on fields caused by the lattice. In Ref. [12] were found that motion of carriers in graphene might be described by Dirac's like equation. Therefore, we met a new type of kinematic properties when we study graphene carriers. In Ref. [13] the Coulomb interaction were added in Hamiltonian for graphene electrons. In this paper we consider the short range interaction (SRI) between excitons in graphene and we suppose that the graphene excitons has the same kinematic properties as the graphene carriers.

Dirac's equation describe the relativistic motion of electron in external electromagnetic field. This equation solution for energy eigenvalues give us solutions with positive and negative energy separated by energy gap. Solution with the negative energies corresponds to the antiparticle states; i.e. to the positron states. When we have deal with the semiconductors we work with the electron's and hole's. Holes are the quasi-particles corresponds to the motion "ionized" states of atoms in semiconductor. To movement of hole corresponds the movement of bound electrons in opposite direction. Electrons of conductivity moving in the space between atoms as quasi-free particles. To the hole motion correspond exchange of electron from one bound state to another between neighbor atoms. Thus, mecha-

nisms of described two types of motion are different. This leads to the different effective mass of electrons and holes, and it's difference from mass of the free electron.

In the literature there is the analogy between behavior of relativistic electrons (the picture of electron-positron states) and electrons in semiconductors (the picture of conductivity electrons and holes). Electrons and positrons has the same mechanism of motion. Difference in the mechanisms of motion and in the effective masses of the electron's and hole's lead to the fact that using of Dirac's equation for description of the electrons with the positive and negative (hole) energy level is not always suitable for semiconductors.

Nevertheless, massless Dirac's like spinor equation has been used for the description of the conductivity electrons in graphene

$$i\hbar\partial_t\psi = \left(\sum_i \left(v_F \sigma_i^\alpha D_i^\alpha + e_i \varphi_{i,ext} \right) + \sum_{i,j \neq i} \frac{1}{2} e_i e_j G_{ij} \right) \psi \quad (1)$$

where ψ is the N -particle wave function which depends on $2N$ coordinate, because graphene is the two-dimensional (2D) structure, v_F is the Fermi velocity of conductivity electrons in graphene, e_i electric charge, σ^α is the spin 1/2 Pauli matrixes, in this equation index α attain to x and y , $\mathbf{D}_i = -i\hbar\nabla_i - e_i/c \cdot \mathbf{A}_{i,ext}$, $\varphi_{i,ext}$ and $\mathbf{A}_{i,ext}$ are the scalar and vector potential of external electromagnetic field, $G_{ij} = 1/r_{ij}$ is the Green function of Coulomb interaction. The same equation might be used for description of the holes in graphene.

Electrons and holes are the charge carriers in graphene. They can form bound states - excitons. Typical velocity of electrons and holes in graphene is the Fermi velocity. The mechanisms of motion of electrons and holes in semiconductors are different, but the typical velocity of they motion in graphene is the same. Thus, we can suppose that center of mass of excitons in graphene move also with the Fermi velocity. Important difference of excitons from electrons and holes it is the fact that spin of exciton equal to integer number of Plank constant: 0 or \hbar . Exciton are Bose particles, whereas electrons and holes are Fermi particles.

As the atoms in quantum gases as the excitons in semiconductors attach a lot attention in the connection with the

Bose-Einstein condensation. For the first time BEC was realized in vapors alkali atoms in 1995. In atomic gases the BEC state reached at hundreds of nanokelvin, whereas in exciton systems BEC state might be realized at several kelvins [3], [14], [15], [16].

We suppose it is possible to realize the BEC of excitons in graphene and suggest model Dirac's like equation for the neutral particles with short range interaction for description of excitons in graphene. This equation is an analog of the equation suggested in Ref.s [12], [13]. Here we present some notice about using of Dirac's like equation for Bose particles description. Used in Ref. [13] equation is linear on momentum operator as Dirac equation. Instead of scalar product of Dirac's matrixes on four momentum this equation contain the scalar product of Pauli matrixes $[\sigma_x, \sigma_y]$ on momentum operator $\mathbf{p} = [p_x, p_y]$. Thus, the model Dirac's like equation used in Ref. [13] might be considered as an analog of spinor Pauli equation containing another Dirac's like kinematic. Consequently, such model equation as the Pauli equation might be used for the description as Bose as Fermi particles.

In this letter we found the general equation for the collective motion of the system of neutral spin-1 particles with SRI whose kinematic properties is analogous to graphene carriers. From this general equation we derive equation for BEC in described particles system. For derivation of equations described collective motion from the many-particle Dirac's like equation we use the method of many-particle quantum hydrodynamics (MPQHD). This method was suggested in 1999 by L. S. Kuz'menkov and S. G. Maksimov [17]. Further development of the MPQHD was made in Ref.s [18]- [24] for various system of particles: quantum plasma [17], [18], [21], particularly for spinning particles [18], [21], [22]; BEC and ultracold fermions of neutral atoms [19]; charged and neutral particles with electric dipole moment [20], particularly electrically polarized BEC [23]; and electrons in graphene [24]. M. Marklund and G. Brodin suggested another way of derivation of QHD equations for spinning particles [25], [27].

The QHD equations as a classic hydrodynamics are very useful for collective excitation studying [20], [21], [23], [25], [28]. In this paper we consider the spectrum of collective excitations in BEC of excitons in graphene.

In this letter we do not present the detail of derivation of QHD equation for excitons BEC in graphene (GEBEC). The general scheme of QHD equations derivation is presented in Ref.s [19], [20], and [24].

We consider BEC in 2D system of particles. BEC cannot take place in a purely 2D system at finite temperatures. Consequently this term may be used for polaritons under very special circumstances (see [29] and ref.s where). Detailed discussion of the superfluidity phenomena and BEC in 2D system is presented in Ref. [30]. Influence of dipolar nature of excitons on one collective properties is also studied where.

For exciton-polariton BEC description the Gross-

Pitaevskii equation is used [31] as for exciton-polaritons dynamic in non-condensed states [32]. Gross-Pitaevskii equation is usually used for studying of BEC in atomic vapors [1]. Gross-Pitaevskii equation is the non-linear Schrodinger equation. This equation has form of the Schrodinger equation for one particle in external field, but also contains the non-linear term proportional to the third degree of the wave function. Gross-Pitaevskii equation might be present in the form of two hydrodynamic equations: continuity equation and Euler equation [1]. Method of QHD allows to derive the Gross-Pitaevskii equation and it's nonlocal generalization from many-particle Schrodinger equation [19].

We present a basic equation here for evolution description of excitons in graphene. Using this equation we derive and present below the equations for description of excitons collective motion and especially for the dynamics of GEBEC.

$$i\hbar\partial_t\psi = \left(\sum_i \left(v_F \hat{s}_i^\alpha p_i^\alpha + V_{i,ext} \right) + \frac{1}{2} \sum_{i,j \neq i} U_{ij} \right) \psi, \quad (2)$$

this equation differ from (1) by the form of spin matrix and the form of interaction. Equation (2) contains following quantities: wave function $\psi = \psi(R, t)$, R is the whole particles coordinates $R = [\mathbf{r}_1, \dots, \mathbf{r}_i, \dots, \mathbf{r}_N]$, $\mathbf{r}_i = [x_i, y_i]$, \hat{s}_i^α are the spin-1 matrixes for i -th particle, $p_i^\alpha = -i\hbar\nabla$ is the momentum operator, $V_{i,ext}$ is the potential of external field, U_{ij} is the SRI potential describing the interaction between excitons in graphene. We notice that at the same time some particle might interact with several particles, by means SRI potential U_{ij} . For this statement illustration we refer to the liquid where molecules is neutral and interacts with the several neighbor molecules. We consider spin-1 particles and spin operators are 3×3 matrixes

$$\hat{s}_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{s}_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix},$$

$$\hat{s}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

the commutation relation for spin-1 matrixes is

$$[\hat{s}_i^\alpha, \hat{s}_j^\beta] = i\delta_{ij}\epsilon^{\alpha\beta\gamma}\hat{s}_i^\gamma. \quad (3)$$

Equation (2) describes the 2D motion of excitons, thus Hamiltonian contains operators $\hat{S}_i^x, \hat{S}_i^y, \hat{p}_i^x$ and \hat{p}_i^y only, but at using commutation relation (3) during equations derivation the \hat{S}_i^z -operator is also appearing.

Graphene is the 2D structure and electrons of graphene are located in the plane. As we describe above in 2D case the electrons has two coordinate x and y , but spin of electrons can be directed in all direction, particularly, in z axes direction, perpendicular to the graphene plane. This fact is accounted by formula (3).

The first equation of the QHD equations set is the continuity equation

$$\partial_t n(\mathbf{r}, t) + v_F \nabla \mathbf{S}(\mathbf{r}, t) = 0. \quad (4)$$

The definition of probability density of conduction electron system in physical space is

$$n(\mathbf{r}, t) = \sum_s \int dR \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \psi^*(R, t) \psi(R, t) \quad (5)$$

where $dR = \prod_{p=1}^N d\mathbf{r}_p$. The continuity equation appears at differentiation of the concentration (5) with respect to time and using equation (2) corresponding to the QHD method.

The quantity $n(\mathbf{r}, t)$ can be considered as 2D excitons concentration. The time evolution of concentration The quantity $\mathbf{S}(\mathbf{r}, t)$ describe the spin density of the system of particles. Coordinate vector \mathbf{r} has only two component. Consequently, equation (4) contains two component of spin density vector \mathbf{S} , these are S_x and S_y . Our next step in construction of the model of collective motion is obtaining of equation of spin evolution. For this aim we differentiate quantity $\mathbf{S}(\mathbf{r}, t)$ with respect to time and use equation (2). Because we known the third component of spin density vector we can study the evolution of whole component of this vector. Therefore, we derive evolution equation for $\mathbf{S} = [S_x, S_y, S_z]$. On this way we have equation of spin evolution:

$$\begin{aligned} \partial_t S^\alpha(\mathbf{r}, t) + v_F \partial^\beta \left(\frac{S^\alpha(\mathbf{r}, t) S^\beta(\mathbf{r}, t)}{n(\mathbf{r}, t)} \right) \\ = -\frac{1}{\hbar} \varepsilon^{\alpha\beta\gamma} J_M^{\beta\gamma}(\mathbf{r}, t) \end{aligned} \quad (6)$$

Here new physical quantity is appeared: $J_M^{\alpha\beta}(\mathbf{r}, t)$. This is the tensor of spin current.

$$\begin{aligned} \partial_t J_M^{\alpha\beta}(\mathbf{r}, t) + \frac{v_F}{\hbar} \partial^\gamma \left(\frac{S^\alpha(\mathbf{r}, t) S^\beta(\mathbf{r}, t) J^\gamma(\mathbf{r}, t)}{n^2(\mathbf{r}, t)} \right) = \\ + \frac{v_F^2}{\hbar} \varepsilon^{\alpha\mu\nu} S^\nu(\mathbf{r}, t) \left(\frac{J^\mu(\mathbf{r}, t) J^\beta(\mathbf{r}, t)}{n^2(\mathbf{r}, t)} - \hbar^2 \frac{\partial^\mu \partial^\beta \sqrt{n(\mathbf{r}, t)}}{\sqrt{n(\mathbf{r}, t)}} \right) \\ - \int d\mathbf{r}' (\partial^\beta U(\mathbf{r}, \mathbf{r}')) j_2^\alpha(\mathbf{r}, \mathbf{r}', t) - v_F S^\alpha(\mathbf{r}, t) \partial^\beta V_{ext}(\mathbf{r}, t). \end{aligned} \quad (7)$$

Equation (7) contains interaction and a new quantity $J^\alpha(\mathbf{r}, t)$, $J^\alpha(\mathbf{r}, t)$ is the probability current.

$$\begin{aligned} \partial_t J^\alpha(\mathbf{r}, t) + v_F \partial^\beta J_M^{\beta\alpha} = -v_F \int d\mathbf{r}' (\partial^\beta U(\mathbf{r}, \mathbf{r}')) \times \\ \times n_2^\alpha(\mathbf{r}, \mathbf{r}', t) - v_F n(\mathbf{r}, t) \partial^\alpha V_{ext}(\mathbf{r}, t). \end{aligned} \quad (8)$$

The equations (7) and (8) contains two-particle function.

These two-particle function appear in the terms contains SRI. In Ref. [19] was developed the method of calculation of the this kind quantities. Using two condition: particles interact via SRI and particles being in the BEC state,

we obtain closed QHD equations set. We do not stop here on the technic of many-particles function calculation. For neutral particles being in the BEC state this technic was described in the Ref. [19]. Here we present the resulting equation found at the condition that excitons are in the BEC state.

$$\partial_t J^\alpha + v_F \partial^\beta J_M^{\beta\alpha} = v_F \Upsilon_{2D} n \partial^\alpha n - v_F n \partial^\alpha V_{ext} \quad (9)$$

and

$$\begin{aligned} \partial_t J_M^{\alpha\beta} + \frac{v_F}{\hbar} \partial^\gamma \left(\frac{S^\alpha S^\beta J^\gamma}{n^2} \right) = \frac{1}{2} v_F \Upsilon_{2D} \partial_\beta (S^\alpha n) \\ + \frac{v_F^2}{\hbar} \varepsilon^{\alpha\mu\nu} S^\nu \left(\frac{J^\mu J^\beta}{n^2} - \hbar^2 \frac{\partial^\mu \partial^\beta \sqrt{n}}{\sqrt{n}} \right) - v_F S^\alpha \partial^\beta V_{ext}. \end{aligned} \quad (10)$$

In these formulas we do not present the arguments of functions.

$$\Upsilon = \pi \int dr(r)^2 \frac{\partial U(r)}{\partial r}, \quad (11)$$

this quantity is the analog of the interaction constant in Gross-Pitaevskii equation [1], [19]. In the BEC theory the SRI interpret via scattering, and $\Upsilon = -4\pi\hbar^2 a/m$, where a is the scattering length in first Born approximation [1]. In Ref. [23] were presented that at derivation of QHD equations we do not consider the scattering approximation of interaction and connection of Υ and scattering length a presented here for handy comparison.

We can analyze the linear dynamics of elemental excitations in the polarized BEC using the QHD equations (4), (6), (9) and (10). The system is placed in an external magnetic field $\mathbf{B}_0 = B_0 \mathbf{e}_z$. The values of concentration n_0 and polarization $\mathbf{S}_0 \parallel \mathbf{B}_0$ for the system in an equilibrium state are constant and uniform and its velocity field $J^\alpha(\mathbf{r}, t)$ and tensor $J^{\alpha\beta}(\mathbf{r}, t)$ values are zero.

We consider the small perturbation of equilibrium state

$$\begin{aligned} n = n_0 + \delta n, \quad S^\alpha = S_0^\alpha + \delta S^\alpha, \\ J^\alpha = 0 + \delta J^\alpha, \quad J^{\alpha\beta} = 0 + \delta J^{\alpha\beta}. \end{aligned} \quad (12)$$

Substituting these relations into system of equations (4), (6), (9) and (10) and neglecting nonlinear terms, we obtain a system of linear homogeneous equations in partial derivatives with constant coefficients. Passing to the following representation for small perturbations δf

$$\delta f = f(\omega, \mathbf{k}) \exp(-i\omega t + i\mathbf{k}\mathbf{r})$$

yields the homogeneous system of algebraic equations. The spin density strength is assumed to have a nonzero value. Expressing all the quantities entering the system of equations in terms of the spin density, we come to the equation

$$\Lambda^{\alpha\beta}(\omega, \mathbf{k}) S^\beta(\omega, \mathbf{k}) = 0, \quad (13)$$

Excitations exist in the case when matrix equation (13) has nontrivial solutions. The condition of nontrivial solution

existence for homogeneous linear algebraical equation set is the determinant of this equation set must be equal to zero:

$$\det \parallel \Lambda^{\alpha\beta}(\omega, \mathbf{k}) \parallel = 0. \quad (14)$$

Solving this equation with respect to ω^2 we obtain a following results.

Equation (14) give us only one wave solution, dispersion of this wave is obtained in the form:

$$\omega^2 = \frac{v_F n_0 \mid \Upsilon \mid k}{\hbar \sqrt{2}}, \quad (15)$$

where $k^2 = k_x^2 + k_y^2$.

Solution (15) does not depend on the sign of SRI, and contain module of the SRI constant Υ .

For comparison we present here the dispersion of the Bogoliubov's mode which is the eigenwave in usual BEC:

$$\omega^2 = \frac{\hbar^2}{4m^2} k^4 + g \frac{n_0}{m} k^2, \quad (16)$$

in the long wavelength limit from formula (16) we have $\omega^2 = g n_0 k^2 / m$. The quantity g is the interaction constant in Gross-Pitaevskii equation [1] and $g = -\Upsilon$. This solution exist for repulsive interaction ($g > 0, a > 0$) only.

Thus, instead of sound like spectrum $\omega \sim k$ for BEC in atoms vapors for the GEBEC we have $\omega \sim \sqrt{k}$.

In connection with the studying developments of the BEC in excitons system in semiconductors and wide using of graphene in semiconductor heterostructures we suggest the model for description of GEBEC collective properties. One of the fundamental properties of many-particle systems is the spectrum of collective excitation. Thus, we considered the collective excitation spectrum for GEBEC.

Using the idea that excitons in graphene has an analogous kinematic properties as the graphene carriers we formulated many-particle Dirac's like equation (or linear on momentum Pauli like equation) for spin-1 neutral particles with SRI, describing graphene excitons dynamics. Starting from this equation, using method of many-particle QHD we derived the system of QHD equations: particles number balance equation (continuity equation), spin balance equation, current evolution equation (Euler equation) and equation evolution of spin current. We considered graphene excitons being in the BEC state and studied the dispersion of collective excitations where. We found that derived model leads to existence of one type of collective excitations in GEBEC. Dispersion dependence of these excitations differ from Bogoliubov's spectrum, which suitable for BEC of atomic vapors or liquid helium. In the case GEBEC the frequency of collective excitations is proportional to \sqrt{k} . Thus, obtained dependence differ from sound wave and long wave length limit of Bogoliubov's spectrum.

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